Lecture 24

Wednesday, December 4, 2019 5:42 AM

Recall: • FEP-homotopy, $\gamma_0 \underset{FEP}{\sim} \gamma_1$.

• Independence of Path Thm from Lecture 23 notes.

• Counting zeros from Lecture 23 notes

Open Mapping Thm. Let $G \subseteq \mathbb{C}$ region, and f anal. and nonconstant in G. Then, $f(U)$ is open for every open $U \subseteq G$.

Pf. Pick $\alpha \in f(U)$, and $a \in U$ s.t. $f(a) = \alpha$. Since f noncanstant, $f(z) - \alpha$ has a zero of finite multi m at a. By previous Thm, $\exists\, B(a,\delta) \subseteq U$ and $B(\alpha, \varepsilon)$ s.t. $f(z) = \beta$ has m simple roots in $B(a,\delta)$ for each $\beta \in B(\alpha,\varepsilon)$. In particular, $B(\alpha,\delta) \subseteq f(U) \Rightarrow f(U)$ open. ☒

Cor. If f is anal. and 1:1 in G, then $\Omega = f(G)$ is open and $f^{-1}: \Omega \to G$ is anal.

Pf. Prevous Prop $\Rightarrow$ if f^{-1} is cont. and $f' \neq 0$ in G, then f^{-1} is anal. w/ $(f^{-1})'(z) = \dfrac{1}{f'(f^{-1}(z))}$.

Well, OM Thm $\Rightarrow f^{-1}$ is cont. Moreover, previous thm shows that if $f'(a) = 0$, $f(a) = \alpha$, then f is, at least, 2:1 near a. Since f is globally 1:1 in G, by assumption, we conclude $f' \neq 0$ in G. Thus, f^{-1} is anal. and $(f^{-1})'(z) = 1/f'(f^{-1}(z))$.